Regularized Grad equations for multicomponent plasmas

Thierry E. Magin*, Gérald Martins* and Manuel Torrilhon†

*Aeronautics and Aerospace Department, von Karman Institute for Fluid Dynamics,
Chaussée de Waterloo 72, 1640 Rhode-Saint-Genèse, Belgium
†Seminar for Applied Mathematics, Swiss Federal Institute of Technology Zurich,
ETH - Zentrum, 8092 Zurich, Switzerland

Abstract. The moment method of Grad is used to derive macroscopic conservation equations for multicomponent plasmas for small and moderate Knudsen numbers, accounting for the electromagnetic field influence and thermal nonequilibrium. In the low Knudsen number limit, the equations derived are fully consistent with those obtained by means of the Chapman-Enskog method. In particular, we have retieved the Kolesnikov effect coupling electrons and heavy particles in the case of the Boltzmann moment systems. Finally, a regularization procedure is proposed to achieve continuous shock structures at all Mach numbers.

Keywords: Kinetic theory; Boltzmann equation; atmospheric entry plasmas; magnetized plasmas; multicomponent diffusion; thermal nonequilibrium; Boltzmann moment systems; Grad equations
PACS: 47.45.Ab; 52.30.Ex

INTRODUCTION

The fluid dynamical description of plasmas can be extended toward the rarefied regime by taking a finite sequence of moments of the Boltzmann equation together with a closure assumption. In this work, we apply the moment method of Grad [1] to derive macroscopic conservation equations for multicomponent plasmas for small and moderate Knudsen numbers, as an alternative to hybrid kinetic-continuum models developed for the transition regime [2]. Given the strong disparity of mass between the electrons and heavy particles (atoms and molecules, neutral or ionized), we conduct a dimensional analysis of the Boltzmann equation following Petit and Darrozes [3] and introduce a multiscale treatment based on two perturbation parameters: the mass parameter, defined as the square root of the ratio of the electron mass to a characteristic heavy-particle mass, and the Knudsen number. The mass parameter governs thermal nonequilibrium between the velocity distribution functions for the electrons and heavy particles; the Knudsen number is associated, as usual, with rarefied gas effects. The low-order moment equations are conservation equations for mass, momentum, and energy at the macroscopic level. They are based on the space of collisional invariants for the Chapman-Enskog method, to obtain Grad equations consistent in the small Knudsen number limit with the Navier-Stokes equations. In this method, the collision operators are linearized in the Knudsen number and the crossed collision operators for electron heavy-particle interactions are expanded in the mass parameter based on two theorems [4]. The collisional invariants are identified from the kernel of these operators. Depending on the type of species, the quasi-equilibrium solutions are Maxwell-Boltzmann velocity distribution functions at either the electron temperature or the heavy-particle temperature. Thermal nonequilibrium is described based on two energy equations for the heavy-particle manifold and the electrons, as opposed to a description based on energy equations for each species of the gas, as proposed by Zhdanov [5]. In the Chapman-Enskog method, the mass and heat fluxes are known to be proportional to diffusion forces according to the Onsager reciprocal relations that are symmetry constraints holding between the transport coefficients. The Kolesnikov effect [6] is a crossed contribution to the mass and energy transport fluxes for multicomponent and partially ionized plasmas, coupling the electrons and heavy particles. This coupling degenerates and vanishes in the case of fully ionized plasmas [7]. Additional terms for the energy exchange between both types of particles are associated with this effect; they are essential to derive a total energy equation and an entropy equation that satisfy the first and second laws of thermodynamics, respectively [4, 8]. Heuristic scalings, such as proposed by [9, 10, 11], do not yield macroscopic equations including the Kolesnikov effect. The Boltzmann moment systems derived in this work include suitable energy transfer terms in the Grad equations for electron and heavy-particle energy conservation, as opposed to previous formulations [5]. Finally, the Grad equations are known to yield unphysical discontinuous shocks for large enough Mach numbers owing to their hyperbolic character. We propose to apply the regularization of Struchtrup and Torrilhon [12, 13].

BOLTZMANN EQUATION

Assumptions

The plasma is composed of electrons, denoted here by the index e, and heavy particles (atoms and molecules, neutral or ionized), denoted by the set of indices H; the full mixture of species is denoted by the set $\mathrm{S} = \{\mathrm{e}\} \cup \mathrm{H}$. The proposed model for multi-component plasmas is based on kinetic theory and classical mechanics. First, we recall that the ratio of the electron mass m_e^0 to a characteristic heavy-particle mass m_h^0 is such that the non-dimensional number $\varepsilon = (m_\mathrm{e}^0/m_h^0)^{1/2}$ is small. The model relies on the following set of assumptions:

1. The reactive collisions and particle internal energy are not accounted for.
2. The inert particle interactions are binary encounters modeled by means of a Boltzmann collision operator.
3. The reference electrical and thermal energies of the system are of the same order of magnitude.
4. The pseudo Mach number, defined as a reference hydrodynamic velocity divided by the heavy-particle thermal speed, $M_h = v^0/V_h^0$, is supposed to be of order one.
5. The macroscopic length scale is based on a reference convective length $L^0 = v^0 t^0$.
6. The reference differential cross-section σ^0 is common to all collisions.

Dimensional analysis and scaling

A sound scaling of the Boltzmann equation is deduced from a dimensional analysis following [3]. Reference dimensional quantities, denoted by the superscript ^{0}, are introduced in Table 1. The characteristic temperature, number density, differential cross-section, mean free path, macroscopic time scale, hydrodynamic velocity, macroscopic length, electric and magnetic fields, and electrical charge are assumed to be common to all species. The mass parameter

$$\varepsilon = \sqrt{m_\mathrm{e}^0/m_h^0}$$

quantifies the ratio of the electron mass to a reference heavy-particle mass. The value of ε being small, electrons exhibit a larger thermal speed than that of heavy particles

$$V_\mathrm{e}^0 = \sqrt{\frac{\mathrm{k_B} T^0}{m_\mathrm{e}^0}}, \qquad V_h^0 = \sqrt{\frac{\mathrm{k_B} T^0}{m_h^0}} = \varepsilon V_\mathrm{e}^0, \tag{1}$$

where $\mathrm{k_B}$ is Boltzmann's constant. Due to their mass disparity, the electrons and heavy particles may have different temperatures, provided that eq. (1) does not fail to describe the order of magnitude of the thermal speeds. The differential cross-sections are assumed to be of the same order of magnitude σ^0. The characteristic mean free path $l^0 = 1/(n^0\sigma^0)$ is found to be identical for all species. Hence, the kinetic time scale, or relaxation time of a distribution function towards its respective quasi-equilibrium state, is lower for electrons than for heavy particles

$$t_\mathrm{e}^0 = \frac{l^0}{V_\mathrm{e}^0}, \qquad t_h^0 = \frac{l^0}{V_h^0} = \frac{t_\mathrm{e}^0}{\varepsilon}. \tag{2}$$

The macroscopic temporal and spatial scales are linked by the relation $L^0 = v^0 t^0$, where the hydrodynamic velocity is determined by the pseudo Mach number $M_h = v^0/V_h^0$, assumed to be of order one. Hence, the Knudsen number is given by the expression

$$Kn = \frac{l^0}{L^0} = \frac{1}{M_h}\frac{t_h^0}{t^0}. \tag{3}$$

When the heavy-particle kinetic time scale is of the order of the macroscopic time scale multiplied by ε, the Knudsen number is $Kn = \varepsilon/M_h$. In this case, the Chapman-Enskog expansion is used to derive macroscopic equations [4, 14, 7]. In this work, we will extend the continuum description to small and moderate values of the Knudsen number by means of the Grad method. Finally, the reference electrical energy is assumed to scale as the thermal energy, $q^0 E^0 L^0 = \mathrm{k_B} T^0$, and the intensity of the magnetic field is governed by the Hall numbers for the electrons and heavy particles

$$\beta_\mathrm{e} = \frac{q^0 B^0}{m_\mathrm{e}^0} t_\mathrm{e}^0 = \varepsilon^{1-b}, \qquad \beta_h = \frac{q^0 B^0}{m_h^0} t_h^0 = \varepsilon \beta_\mathrm{e}, \tag{4}$$

TABLE 1. Reference quantities used in the dimensional analysis.

	Common to all species	
Temperature	T^0	
Number density	n^0	
Differential cross-section	σ^0	
Mean free path	l^0	
Macroscopic time scale	t^0	
Hydrodynamic velocity	v^0	
Macroscopic length	L^0	
Electric field	E^0	
Magnetic field	B^0	
Electron charge absolute value	q^0	
	Electrons	**Heavy particles**
Mass	m_e^0	m_h^0
Thermal speed	V_e^0	V_h^0
Kinetic time scale	t_e^0	t_h^0

defined as the Larmor frequencies, q^0B^0/m_e for the electrons and q^0B^0/m_h^0 for the heavy particles, multiplied by their respective kinetic time scale. The magnetic field is assumed to be proportional to a power of ε by means of an integer b that characterizes its intensity: $b<0$ for unmagnetized plasmas, $b=0$, for weakly magnetized plasmas, and $b=1$ for strongly magnetized plasmas [4]. We investigate the system at the macroscopic time t^0 and macroscopic length L^0 by means of the Boltzmann equation expressed in nondimensional form in the heavy-particle velocity v_h reference frame, as

$$\partial_t f_\mathrm{e} + \tfrac{1}{\varepsilon M_h}(C_\mathrm{e}+\varepsilon M_h v_h)\cdot\partial_x f_\mathrm{e} + \frac{\varepsilon^{-b}}{M_h Kn} q_e\big[(C_\mathrm{e}+\varepsilon M_h v_h)\wedge B\big]\cdot\partial_{C_\mathrm{e}} f_\mathrm{e} + \big(\tfrac{1}{\varepsilon M_h} q_e E - \varepsilon M_h \tfrac{\mathrm{D}v_h}{\mathrm{D}t}\big)\cdot\partial_{C_\mathrm{e}} f_\mathrm{e} - (\partial_{C_\mathrm{e}} f_\mathrm{e}\otimes C_\mathrm{e}):\partial_x v_h = \tfrac{1}{\varepsilon M_h Kn}\big[\mathfrak{J}_\mathrm{ee}(f_\mathrm{e},f_\mathrm{e}) + \sum_{j\in\mathrm{H}} \mathfrak{J}_{\mathrm{e}j}(f_\mathrm{e},f_j)\big], \quad (5)$$

$$\partial_t f_i + \tfrac{1}{M_h}(C_i+M_h v_h)\cdot\partial_x f_i + \frac{\varepsilon^{2-b}}{M_h Kn}\tfrac{q_i}{m_i}\big[(C_i+M_h v_h)\wedge B\big]\cdot\partial_{C_i} f_i + \big(\tfrac{1}{M_h}\tfrac{q_i}{m_i}E - M_h\tfrac{\mathrm{D}v_h}{\mathrm{D}t}\big)\cdot\partial_{C_i} f_i - (\partial_{C_i} f_i\otimes C_i):\partial_x v_h = \tfrac{1}{M_h Kn}\big[\tfrac{1}{\varepsilon}\mathfrak{J}_{i\mathrm{e}}(f_i,f_\mathrm{e}) + \sum_{j\in\mathrm{H}}\mathfrak{J}_{ij}(f_i,f_j)\big], \quad i\in\mathrm{H}, \quad (6)$$

for the electron and heavy particle velocity distribution functions, f_e and f_i, $i\in\mathrm{H}$, respectively. Symbol $\mathrm{D}/\mathrm{D}t = \partial_t + v_h\cdot\partial_x$ stands for the material derivative. The choice of the heavy-particle velocity frame is natural for plasmas: it is the reference frame in which the heavy particles thermalize and in which all particles diffuse [4]. The multiscale analysis occurs at three levels: a) in the kinetic eqs. (5) and (6) in terms of Kn and ε; b) in the crossed collision operators $\mathfrak{J}_{\mathrm{e}i}$ and $\mathfrak{J}_{i\mathrm{e}}$, $i\in\mathrm{H}$, in terms of ε; c) in the collisional invariants in terms of ε and thus in the conservation of the associated macroscopic quantities. Encounters between particles of the same type, $\mathfrak{J}_\mathrm{ee}$ and $\mathfrak{J}_{ij}$, $i,j\in\mathrm{H}$, are dealt with as usual, whereas the collision operators between unlike particles (electron heavy-particle interactions) depend on the ε parameter via their relative kinetic energy, velocity, and deflection angle. These collision operators are expanded in this parameter, as follows [4]

Theorem 1 *The collision operators $\mathfrak{J}_{i\mathrm{e}}$ and $\mathfrak{J}_{\mathrm{e}i}$, $i\in\mathrm{H}$, can be expanded in the form*

$$\begin{aligned}
\mathfrak{J}_{i\mathrm{e}}(f_i,f_\mathrm{e})(C_i) &= \varepsilon\mathfrak{J}^1_{i\mathrm{e}}(f_i,f_\mathrm{e})(C_i) + \varepsilon^2\mathfrak{J}^2_{i\mathrm{e}}(f_i,f_\mathrm{e})(C_i) + \varepsilon^3\mathfrak{J}^3_{i\mathrm{e}}(f_i,f_\mathrm{e})(C_i) + \mathscr{O}(\varepsilon^4),\\
\mathfrak{J}_{\mathrm{e}i}(f_\mathrm{e},f_i)(C_\mathrm{e}) &= \mathfrak{J}^0_{\mathrm{e}i}(f_\mathrm{e},f_i)(C_\mathrm{e}) + \varepsilon\mathfrak{J}^1_{\mathrm{e}i}(f_\mathrm{e},f_i)(C_\mathrm{e}) + \varepsilon^2\mathfrak{J}^2_{\mathrm{e}i}(f_\mathrm{e},f_i)(C_\mathrm{e}) + \varepsilon^3\mathfrak{J}^3_{\mathrm{e}i}(f_\mathrm{e},f_i)(C_\mathrm{e}) + \mathscr{O}(\varepsilon^4),
\end{aligned}$$

where the zero-order collision operator $\mathfrak{J}^0_{i\mathrm{e}}(f_i,f_\mathrm{e})(C_i)$, $i\in\mathrm{H}$, vanishes.

Graille *et al*. [4] have identified the space of collision invariants for the scaling $Kn = \varepsilon/M_h$ of the Boltzmann equation. In this case, a Chapman-Enskog method can be used and the collision operators are expanded in the Knudsen number for the velocity distribution functions. The space of electron collisional invariants for mass and energy is spanned by

$$\begin{cases} \hat{\psi}_{\mathrm{e}}^{1} \;=\; 1, \\ \hat{\psi}_{\mathrm{e}}^{2} \;=\; \frac{1}{2} C_{\mathrm{e}} \cdot C_{\mathrm{e}}, \end{cases} \tag{7}$$

and the space of heavy-particle collisional invariants for mass, momentum, and energy by

$$\begin{cases} \hat{\psi}_{h}^{j} \quad = \; \left(m_i \delta_{ij} \right)_{i \in \mathrm{H}}, & j \in \mathrm{H}, \\ \hat{\psi}_{h}^{n^{\mathrm{H}}+\nu} = \; \left(m_i C_{i\nu} \right)_{i \in \mathrm{H}}, & \nu \in \{1,2,3\}, \\ \hat{\psi}_{h}^{n^{\mathrm{H}}+4} = \; \left(\frac{1}{2} m_i C_i \cdot C_i \right)_{i \in \mathrm{H}}, \end{cases} \tag{8}$$

where symbol n^{H} denotes the cardinality of the set of heavy particles H. The low-order moment equations derived in this work are based on the collisional invariants obtained by means the Chapman-Enskog method to obtain equations consistent in the low Knudsen number limit. Notice that the peculiar velocities C_i, $i \in \mathrm{S}$, are expressed in the heavy-particle velocity v_h reference frame, hence the global heavy-particle mass flux vanishes, *i.e.*, $\sum\limits_{j \in \mathrm{H}} \rho_j \omega_j^s = 0$.

MODIFIED GRAD-ZHDANOV EQUATIONS

In this section, we review the Grad equations derived in nondimensional form for arbitrary types of interaction potentials used to accurately describe collisions between the particles of multicomponent, unmagnetized, and quasi-neutral plasmas. The terms quadratic in Kn and ε are neglected in this analysis, for simplicity of the formulation.

Mass conservation equation

The species conservation equations for mass densities ρ_i, $i \in \mathrm{S}$, are derived as

$$\frac{\mathrm{D}\rho_{\mathrm{e}}}{\mathrm{D}t} + \rho_{\mathrm{e}} \frac{\partial v_h^s}{\partial x_s} + \frac{1}{M_h} \frac{\partial(\rho_{\mathrm{e}} \omega_e^s)}{\partial x_s} \quad = \quad 0, \tag{9}$$

$$\frac{\mathrm{D}\rho_i}{\mathrm{D}t} + \rho_i \frac{\partial v_h^s}{\partial x_s} + \frac{Kn}{M_h} \frac{\partial(\rho_i \omega_i^s)}{\partial x_s} \quad = \quad 0, \quad i \in \mathrm{H}, \tag{10}$$

with the diffusion velocities ω_i^s, $i \in \mathrm{S}$. Summing up eq. (10) over heavy particles

$$\frac{\mathrm{D}\rho_h}{\mathrm{D}t} + \rho_h \frac{\partial v_h^s}{\partial x_s} = 0, \tag{11}$$

we notice that the heavy-particle mass is conserved in the heavy-particle velocity reference frame

Mass diffusion equation

Based on the ansatz that the electron viscous tensor vanishes, the species conservation equations for the diffusion velocities are given by the relations

$$\frac{\partial p_{\mathrm{e}}}{\partial x_r} - n_{\mathrm{e}} q_e E^r = -\frac{\varepsilon}{Kn} M_h \sum_{j \in \mathrm{H}} n_j F_{je}^r, \tag{12}$$

$$Kn \frac{\mathrm{D}(\rho_i \omega_i^r)}{\mathrm{D}t} + Kn \rho_i \left(\omega_i^r \frac{\partial v_h^s}{\partial x_s} + \omega_i^s \frac{\partial v_h^r}{\partial x_s} \right) + M_h \rho_i \frac{\mathrm{D} v_h^r}{\mathrm{D}t} + \frac{1}{M_h} \left(Kn \frac{\partial \pi_i^{rs}}{\partial x_s} + \frac{\partial p_i}{\partial x_r} - n_i q_i E^r \right)$$
$$= \frac{n_i}{Kn} \left(\varepsilon F_{ie}^r + Kn \sum_j F_{ij}^r \right), \quad i \in \mathrm{H}, \tag{13}$$

with the partial pressures $p_e = n_e T_e$ and $p_i = n_i T_h$, $i \in \mathrm{H}$. In eq. (12), the electron pressure gradient and electric force are related to the momentum exchanged between electrons and heavy particles, that is expressed in terms of the average electron force acting on the heavy particle $i \in \mathrm{H}$, introduced as

$$F_{ie}^{s} = -\frac{16}{3\pi}\frac{n_e}{M_h}\Big[\omega_e^s \Omega_{ie}^{11} + \frac{h_e^s}{p_e}\big(\frac{2}{5}\Omega_{ie}^{12} - \Omega_{ie}^{11}\big)\Big], \tag{14}$$

with the reduced heat flux $h_e^r = q_e^r - \frac{5}{2} p_e \omega_e^r$. Symbols Ω_{ij}^{pq} stand for the transport collision integrals of order p,q for species i,j. The average force due to the heavy particle j acting on the heavy particle i is defined as

$$F_{ij}^{s} = -\frac{16}{3\pi}\frac{n_j}{M_h}\frac{m_i m_j}{m_i+m_j}\Big[(\omega_j^s - \omega_i^s)\Omega_{ij}^{11} + \frac{m_i m_j}{m_i+m_j}\big(\frac{h_j^s}{m_i p_i} - \frac{h_i^s}{m_j p_j}\big)\big(\frac{2}{5}\Omega_{ij}^{12} - \Omega_{ij}^{11}\big)\Big] \tag{15}$$

Summing up eqs. (12) and (13), the momentum conservation is expressed as

$$\rho_h \frac{\mathrm{D} v_h^r}{\mathrm{D}t} + \frac{1}{M_h^2}\Big(Kn \frac{\partial \pi_h^{sr}}{\partial x_s} + \frac{\partial p}{\partial x_r} - nqE^r\Big) = 0, \tag{16}$$

where symbol $\pi_h^{rs} = \sum_{j\in \mathrm{H}} \pi_j^{rs}$ stands for the heavy-particle viscous tensor. The mixture pressure is given by expression $p = \sum_{j\in \mathrm{S}} p_j$ and the total density of charge is given by $nq = \sum_{j\in \mathrm{S}} n_j q_j$.

Viscous stress tensor equation

The species conservation equations for the viscous tensor of the heavy particle $i \in \mathrm{H}$ is expressed as

$$\frac{\mathrm{D}\pi_i^{rs}}{\mathrm{D}t} + \pi_i^{rs}\frac{\partial v_h^t}{\partial x_t} + 2\pi_i^{t\langle s}\frac{\partial v_h^{r\rangle}}{\partial x_t} + 2\frac{p_i}{Kn}\frac{\partial v_h^{\langle r}}{\partial x_{s\rangle}} + \frac{1}{M_h}\frac{4}{5}\frac{\partial q_i^{\langle r}}{\partial x_{s\rangle}} + \frac{1}{KnM_h}\frac{\partial u^0_{i\,rst}}{\partial x_t} + 2M_h\rho_i\omega_i^{\langle r}\Big(\frac{\mathrm{D}v_h^{s\rangle}}{\mathrm{D}t} - \frac{1}{M_h^2}\frac{q_i}{m_i}E^{s\rangle}\Big) = -\bar{\nu}_i \pi_i^{rs}, \tag{17}$$

$i \in \mathrm{H}$, with the high-order moment $u^a_{i,k_1\ldots k_n} = m_i \int C_i^{2a} C_{i\langle k_1}\ldots C_{i k_n\rangle} f_i dC_i$. Quantity q_i^r stands for the heat flux of heavy particle $i\in \mathrm{H}$. The BGK approximation has been used for the collision operator to derive the right-hand-side of eq. (17).

Energy conservation equation

The equations for the electron energy e_e and heavy-particle energy e_h are written as

$$\frac{\mathrm{D}(\rho_e e_e)}{\mathrm{D}t} + \frac{5}{3}\rho_e e_e \frac{\partial v_h^s}{\partial x_s} + \frac{1}{M_h}\frac{\partial q_e^s}{\partial x_s} - \frac{1}{M_h} n_e \omega_e^r q_e E^r = -\Delta E_h^0 - \varepsilon \Delta E_h^1, \tag{18}$$

$$\frac{\mathrm{D}(\rho_h e_h)}{\mathrm{D}t} + \frac{5}{3}\rho_h e_h \frac{\partial v_h^s}{\partial x_s} + \frac{Kn}{M_h}\frac{\partial q_h^s}{\partial x_s} - \frac{Kn}{M_h}\sum_{j\in \mathrm{H}} n_j \omega_j^r q_j E^r + Kn\pi_h^{rs}\frac{\partial v_h^r}{\partial x_s} = \Delta E_h^0 + \varepsilon \Delta E_h^1, \tag{19}$$

with the electron and heavy-particle heat fluxes q_e^s and $q_h^s = \sum_{j\in \mathrm{H}} q_j^r$. The Landau-Teller expression for energy exchange between electrons and heavy particles is given by the expression

$$\Delta E_h^0 = \frac{\varepsilon}{Kn}\frac{16 n_e}{\pi M_h}(T_e - T_h)\sum_{j\in \mathrm{H}} \frac{n_j}{m_j}\Omega_{je}^{11} \tag{20}$$

Quantity $\Delta E_h^1 = -\sum_{j\in \mathrm{H}} n_j \omega_j^s F_{je}^s$ is the Kolesnikov effect contribution. This term is essential to derive a nonegative entropy production rate in the Chapman-Enskog method [4] and thus satisfy the second law of thermodynamics. Adding eqs. (18) and (19) and the momentum eq. (16) multiplied by the heavy-particle velocity, the equation for the total energy $E = \rho e + M_h^2 \frac{1}{2}\rho_h v_h^s v_h^s$ is given by the relation

$$\frac{\mathrm{D}E}{\mathrm{D}t} + E\frac{\partial v_h^s}{\partial x_s} + \frac{\partial}{\partial x_s}(p v_h^s + Kn\pi_h^{rs} v_h^s) + \frac{1}{M_h}\frac{\partial}{\partial x_s}\big(q_e^s + Kn\, q_h^s\big) - \Big(nqv_h^r + \frac{1}{M_h}\big(n_e q_e \omega_e^r + Kn\sum_{j\in \mathrm{H}} n_j q_j \omega_j^r\big)\Big)E^r = 0, \tag{21}$$

hence the first law of thermodynamics is also satisfied.

Reduced heat flux equation

The species conservation equations for the reduced heat fluxes $h_i^r = q_i^r - \frac{5}{2} p_i \omega_i^r$, $i \in \mathrm{S}$, are given by the expressions

$$\frac{\mathrm{D}h_e^r}{\mathrm{D}t} + \frac{7}{5}\frac{\partial v_h^s}{\partial x_s} h_e^r + \frac{7}{5}\frac{\partial v_h^r}{\partial x_s} h_e^s + \frac{2}{5}\frac{\partial v_h^s}{\partial x_r} h_e^s + \frac{1}{\varepsilon}\frac{\partial v_h^s}{\partial x_t} u^0_{e|rst} + 2p_\mathrm{e}\omega_e^s \frac{\partial v_h^{\langle r}}{\partial x_{s\rangle}} + \frac{5}{3} p_\mathrm{e}\omega_e^r \frac{\partial v_h^s}{\partial x_s} + \frac{1}{\varepsilon^2 M_h}\frac{\partial}{\partial x_s}\left[\frac{u^1_{e|rs}}{2}\right]$$
$$+\frac{1}{\varepsilon^2 M_h}\frac{\partial}{\partial x_r}\left[\frac{u_e^2}{6} - \frac{15}{6}\frac{p_\mathrm{e}^2}{\rho_\mathrm{e}}\right] + \frac{5}{2}\rho_\mathrm{e}\omega_e^r \frac{\mathrm{D}}{\mathrm{D}t}\frac{p_e}{\rho_\mathrm{e}} + \frac{1}{\varepsilon^2 M_h}\frac{5}{2} p_\mathrm{e} \frac{\partial}{\partial x_r}\frac{p_e}{\rho_\mathrm{e}} = -\bar{\nu}_e h_e^r \tag{22}$$

$$\frac{\mathrm{D}h_i^r}{\mathrm{D}t} + \frac{7}{5}\frac{\partial v_h^s}{\partial x_s} h_i^r + \frac{7}{5}\frac{\partial v_h^r}{\partial x_s} h_i^s + \frac{2}{5}\frac{\partial v_h^s}{\partial x_r} h_i^s + \frac{1}{Kn}\frac{\partial v_h^s}{\partial x_t} u^0_{i|rst} + 2p_i\omega_i^s \frac{\partial v_h^{\langle r}}{\partial x_{s\rangle}} + \frac{5}{3} p_i\omega_i^r \frac{\partial v_h^s}{\partial x_s}$$
$$+\frac{1}{M_h Kn}\frac{\partial}{\partial x_s}\left[\frac{u^1_{i|rs}}{2} - Kn\frac{5}{2}\frac{p_i}{\rho_i}\pi_i^{rs}\right] + \frac{1}{KnM_h}\frac{\partial}{\partial x_r}\left[\frac{{u_i}^2}{6} - \frac{15}{6}\frac{p_i^2}{\rho_i}\right] + \frac{5}{2}\rho_i\omega_i^r\frac{\mathrm{D}}{\mathrm{D}t}\frac{p_i}{\rho_i} + \frac{1}{M_h}\frac{5}{2}\pi_i^{rs}\frac{\partial}{\partial x_s}\frac{p_i}{\rho_i}$$
$$+\frac{1}{M_h Kn}\frac{5}{2}p_i\frac{\partial}{\partial x_r}\frac{p_i}{\rho_i} + M_h\left(\frac{Dv_h^s}{Dt} - \frac{1}{M_h^2}\frac{q_i}{m_i}E^s\right)\pi_i^{rs} = -\bar{\nu}_i h_i^r, \quad i \in \mathrm{H}, \tag{23}$$

with the high-order moments $u^a_{e,k_1\ldots k_n} = \int {C_\mathrm{e}}^{2a} C_{e_{\langle k_1}} \ldots C_{e_{k_n\rangle}} f_\mathrm{e}\, dC_\mathrm{e}$ and $u^a_{i,k_1\ldots k_n} = m_i \int {C_i}^{2a} C_{i_{\langle k_1}} \ldots C_{i_{k_n\rangle}} f_i\, dC_i$. The BGK approximation has been used for the collision operator to derive the right-hand-side of these equations.

CONCLUSION

The moment method of Grad was used to derive macroscopic conservation equations for multicomponent plasmas for small and moderate Knudsen numbers, accounting for the electromagnetic field influence and thermal nonequilibrium. Struchtrup and Torrilhon [12, 13] have introduced a regularization of the Grad equations, which is non-hyperbolic, and yields continuous shock structures at all Mach numbers. We propose to apply this regularization consisting in creating an expansion about the moment equations, thus allowing small deviations from the assumed distribution function. This procedure results in the addition of elliptic terms to the standard moment equations. For a thorough description of the regularization, the reader is referred to [16].

ACKNOWLEDGMENTS

This work was initiated during a Phase I STTR AF08-T019 grant monitored by Dr. J. Schmisseur. The Institute for Pure and Applied Mathematics is gratefully acknowledged for interactions fostered during the workshop on *"Computational kinetic transport and hybrid methods"* organized at UCLA in 2009.

REFERENCES

1. H. Grad, *Communication on Pure Applied Mathematics*, **2**, 331–407 (1949).
2. T. E. Schwartzentruber and I. .D. Boyd, *Journal of Computational Physics* **215**, 402–416 (2006).
3. J.-P. Petit and J.-S. Darrozes, *Journal de Mécanique* **14(4)**, 745–759 (1975).
4. B. Graille, T. E. Magin, and M. Massot, *Math. Models Methods Appl. Sci.* **19(4)**, 527–599 (2009).
5. V. M. Zhdanov, *Transport processes in multicomponent plasma*, Taylor and Francis, London, 2002.
6. A. F. Kolesnikov, *Technical Report 1556*, Institute of Mechanics, Moscow State University, Moscow, 1974.
7. P. Degond and B. Lucquin-Desreux, *Math. Models Methods Appl. Sci.* **6(3)** 405–436 (1996).
8. V. Giovangigli, *Multicomponent flow modeling*, Birkhäuser, Boston, 1999.
9. S. I. Braginskii, *Soviet Physics JETP* **6**, 358–369 (1958).
10. R. S. Devoto, *Physics of Fluids* **9** 1230–1240(1966).
11. R. M. Chmieleski and J. H. Ferziger, *Physics of Fluids* **10** 2520–2530 (1967).
12. H. Struchtrup and M. Torrilhon, *Physics of Fluids* **15**, 2668–2680 (2003).
13. M. Torrilhon and H. Struchtrup, *Journal of Fluid Mechanics* **513**, 171–198 (2004).
14. T. E. Magin and G. Degrez, *Physical Review E* **70**, 046412 (2004).
15. W. Marques, *Continuum Mech. Thermodyn.* **8**, 53-64 (1996).
16. H. Struchtrup, *Macroscopic transport equations for rarefied gas fows*, Springer, Berlin 2005.